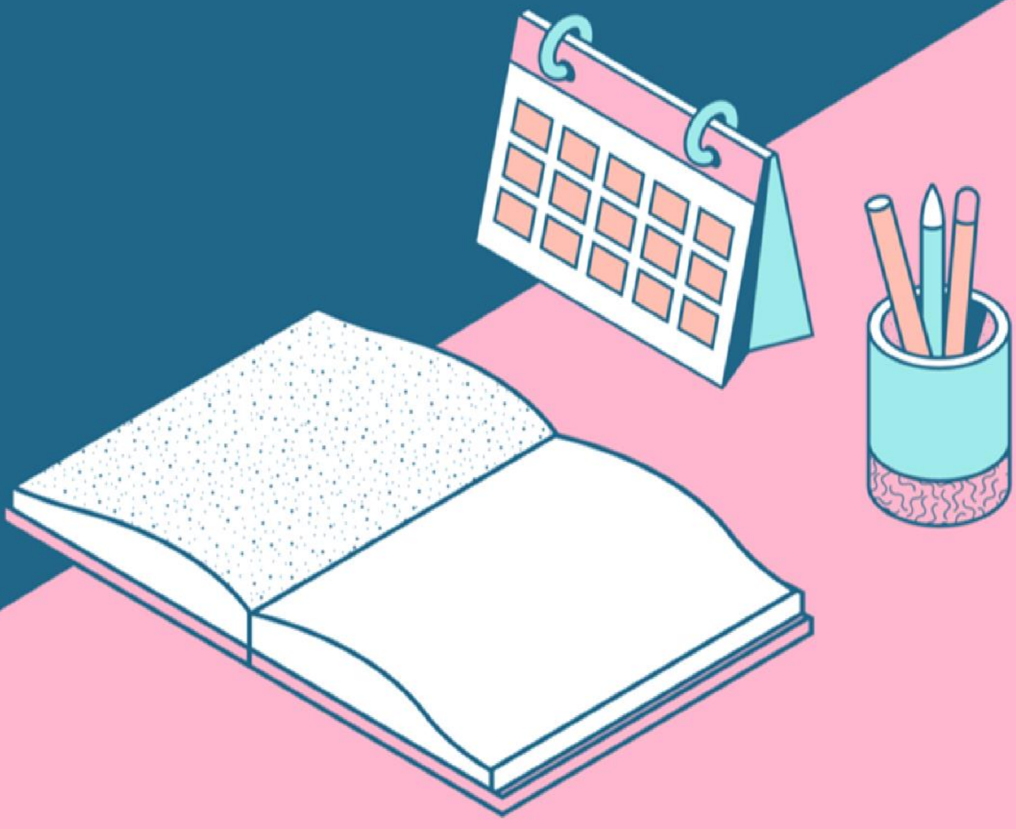
MATHEMATICS FORMULAE

BY

DR. SUHAS PATIL



DERIVATIVE

If $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

is exists, then it is called as derivative of function f

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Rules of Derivative:

1 Addition Rule : $\frac{d}{dx}(u+v) = \frac{d(u)}{dx} + \frac{d(v)}{dx}$

2 Subtraction Rule : $\frac{d}{dx}(u-v) = \frac{d(u)}{dx} - \frac{d(v)}{dx}$

3 Product Rule : $\frac{d}{dx}(u \times v) = u\frac{d(v)}{dx} + v\frac{d(u)}{dx}$

4 Quotient Rule $\frac{d}{dx}\left(\frac{u}{y}\right) = \frac{v \cdot \frac{d}{dx}(u) - u\frac{d}{dx}(v)}{(v)^2}$

Standard Derivative:

1 $\frac{d}{dx}(c) = 0$ (c is constant)

2 $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$

3 $\frac{d}{dx}(e^x) = e^x$, (c is const)

4 $\frac{d}{dx}(a^x) = a^x \log a$

5 $\frac{d}{dx}(\log x) = \frac{1}{x}$

6 $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

Trigonometric Functions:

1 $\frac{d}{dx}(\sin x) = \cos x$

2 $\frac{d}{dx}(\cos x) = -\sin x$

3 $\frac{d}{dx}(\tan x) = \sec^2 x$

4 $\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$

5 $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

6 $\frac{d}{dx}(\mathrm{cosec}\, x) = -\mathrm{cosec}\, x \cdot \cot x$

Inverse Trigonometric Functions:

1 $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

2 $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

3 $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

4 $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

5 $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

6 $\frac{d}{dx}(\text{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$

Hyperbolic Function | **Inverse Hyperbolic**

Hyperbolic Function	Inverse Hyperbolic
1 $\frac{d}{dx}(\sinh x) = \cosh x$	1 $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$
2 $\frac{d}{dx}(\cosh x) = \sinh x$	2 $\cdot \frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$ $(x > 1)$
3 $\frac{d}{dx}(\tanh x) = \text{sech}^2 x$	3. $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$ $(\lvert x\rvert < 1)$
4 $\frac{d}{dx}(\coth x) = -\text{cosech}^2 x$	4 $\cdot \frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$

5 $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \,.\, tanhx$

6 $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{cosech} x \,.\, cothx$

5. $\frac{d}{dx}(\operatorname{sech}^{-1} x) = \frac{-1}{x\sqrt{1-x^2}}$

6. $\frac{d}{dx}(\operatorname{cosech}^{-1} x) = \frac{-1}{|x|\sqrt{1-x^2}}$

$(x \neq 0)$

INTEGRATION

Integration:-

"Integration is opposite process of differentiation

If $\frac{d}{dx}[f(x)] = \mathbf{p}(x)$ then $\int \mathbf{p}(x)dx = f(x) + c$

(c is constant of integration)

Rules of Integration:-

1 **Addition/Subtraction Rule** :

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

2 **Integration by Parts** :

$$\int \left[\mathrm{I}^{st} II^{nd} dx = I^{st} \int II^{nd} dx - \int \left[\frac{d}{dx}(I^{st}) \cdot \int II^{nd} dx\right] dx\right.$$

The first function and second function are taken according to the order of the word LIATE.

$$\text{LATE} = \begin{cases} L \rightarrow \text{ Logaritmic function (eg. log x, log 2x.......)} \\ \text{I} \rightarrow \text{ Inverse trigonometric function (eg. } \sin^{-1} x, \cos^{-1} x \ldots) \\ A \rightarrow \text{ Algebric function (eg. } 3x, \text{ 7x+9.......)} \\ T \rightarrow \text{ Trigonometric function (eg. sin x, tan x, sec x.......)} \\ E \rightarrow \text{ Exponential function } (eg. e^x, e^{2x} \ldots \ldots) \end{cases}$$

Standard Integration:

1 $\int dx = x + c$

2 $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

3 $\int e^x dx = e^x + c$

4 $\int a^x dx = \frac{a^x}{\log a} + c$

5 $\int \frac{1}{x} dx = \log x + c$

6 $\int k dx = kx + c$, (k is const)

7 $\int \sin x \, dx = -\cos x + c$

8 $\int \cos x dx = \sin x + c$

9 $\int \tan x dx = \log(\sec x) + c$

10 $\int \cot x dx = \log(\sin x)$

11 $\int \sec x dx = \log(\sec x + \tan x) + c = \log\left[\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right] + c$

12 $\int \operatorname{cosec} x dx = \log(\operatorname{cosec} x - \cot x) + c = \log\left[\tan\left(\frac{x}{2}\right)\right] + c$

13 $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$ **OR** $-\cos^{-1} x + c$

14 $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$ **OR** $-\cot^{-1} x + c$

15 $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$ **OR** $-\operatorname{cosec}^{-1} x + c$

16 $\int \sec^2 x dx = \tan x + c$

17 $\int \operatorname{cosec}^2 x dx = -\cot x + c$

18 $\int \sec x \tan x dx = \sec x + c$

19 $\int \operatorname{cosec} x \cdot \cot x dx = -\operatorname{cosec} x + c$

20 $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$

21 $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log\left(\frac{x-a}{x+a}\right) + c$

22 $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \log\left(\frac{a+x}{a-x}\right) + c$

23 $\int \frac{1}{\sqrt{x^2+a^2}} dx = \log\left(x + \sqrt{x^2 + a^2}\right) + c$

24 $\int \frac{1}{\sqrt{x^2-a^2}} dx = \log\left(x + \sqrt{x^2 - a^2}\right) + c$

25 $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

26 $\int \sqrt{x^2 + a^2} dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right) + c$

27 $\int \sqrt{x^2 - a^2} dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right) + c$

28 $\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{d^2}{2}\sin^{-1}\left(\frac{x}{2}\right) + c$

29 $\int \sqrt{x^2 + a^2} dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left[x + \sqrt{x^2 + a^2}\right] + c$

30 $\int \sqrt{x^2 - a^2} dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left[x + \sqrt{x^2} - a^2\right] + c$

<u>**Standard Integral Result:**</u>-

1 $\int \frac{f'(x)}{f(x)} dx = \log[f(x)] + c$

2 $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$

3 $\int [f(x)]^n f(x) dx = \frac{[f(x)]^{n+1}}{n+1} + c$

4 $\int e^x [f(x) + f'(x)] dx = e'' f(x) + c$

5 $\int e^{ax}\sin(bx+c)dx = \frac{e^{ax}}{\alpha^2+b^2}[a\sin(bx+c) - b\cos(bx+c)] + c$

6 $\int e^{e^x}\cos(bx+c)dx = \frac{e^{ax}}{\alpha^2+b^2}[a\cos(bx+c) + b\sin(bx+c)] + c$

<u>Integration of Hyperbolic Function:-</u>

1 $\int \sinh x dx = \cosh x + c$

2 $\int \cosh x dx = \sinh x + c$

3 $\int \tanh x dx = \log(\cosh x) + c$

4 $\int \coth x dx = \log(\sinh x) + c$

5 $\int \operatorname{sech} x dx = 2\tan^{-1}(e^x) + c$

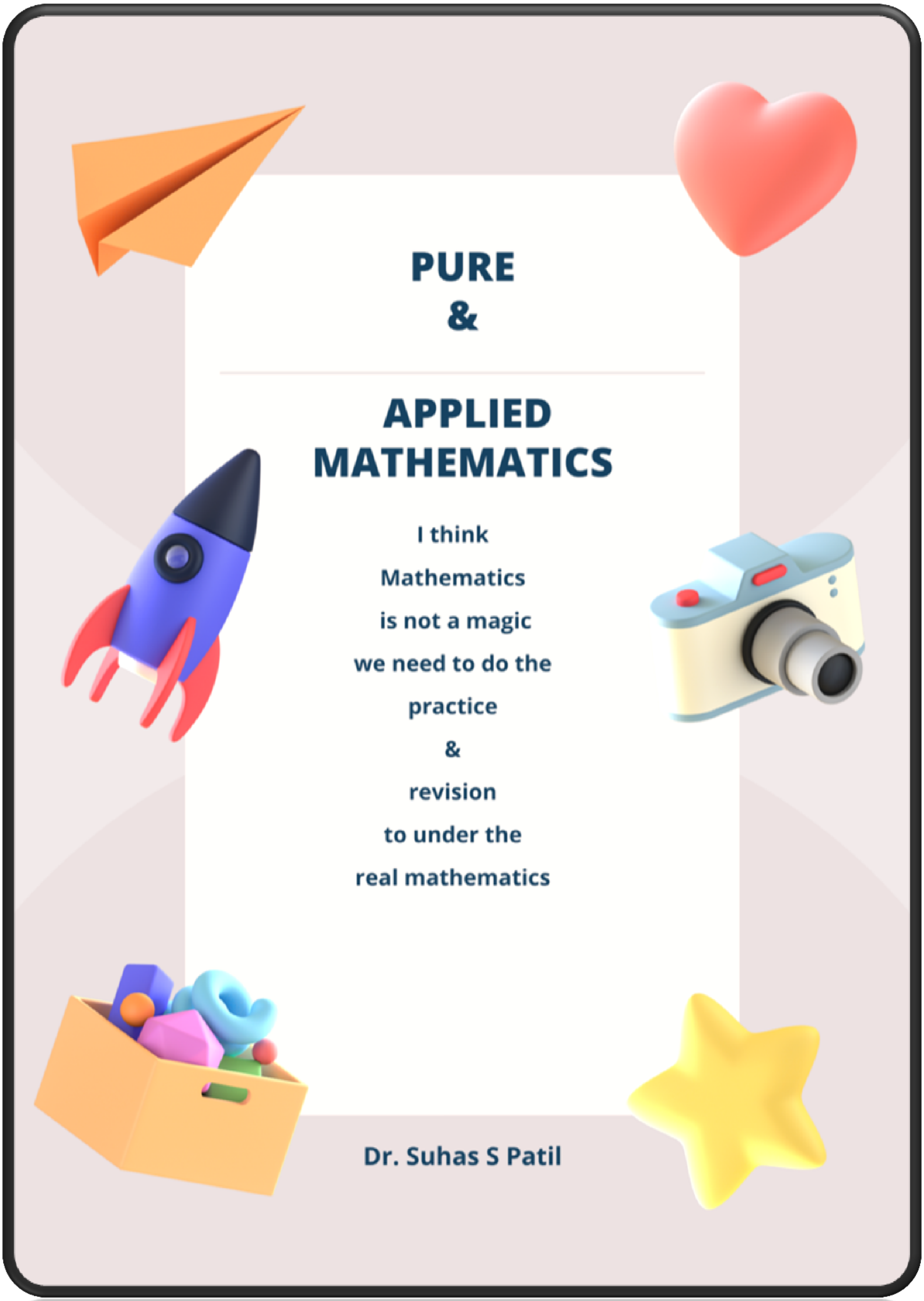
6 $\int \operatorname{cosech} x dx = \log\left(\tanh\frac{\pi}{2}\right) + c$

7 $\int \operatorname{sech}^2 x dx = \tanh x + c$

8 $\int \operatorname{cosech}^2 x dx = -\coth x + c$

9 $\int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + c$

10 $\int \operatorname{cosech} x \cdot \coth x dx = -\operatorname{cosech} x + c$


PURE
&
APPLIED
MATHEMATICS
I think
Mathematics
is not a magic
we need to do the
practice
&
revision
to under the
real mathematics
Dr. Suhas S Patil