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RANDOM FIXED POINT THEOREMS FOR COMPATIBLE MAPPINGS IN METRIC SPACE

[1]SUHAS SHIVAJIRAO PATIL*
[1]Research Scholar, SRTMU, Nanded (M.S), India.

[2]U. P. DOLHARE
[2]Associate Professor, Department of Mathematics, D.S.M. College, Jintur (M.S), India.

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ABSTRACT

"*G. Jungck [1] Introduce the concept of compatible mapping using two self-maps. It is a generalization of commuting maps (i.e. Compatibility implies Poin-twise R-Weakly Commutativity and are characterized in terms of coincidence point). In this article we form a new generalized contractive condition and give some common fixed point theorems for compatible mappings****".***

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I. INTRODUCTION AND PRELIMINARIES

In 1986 Jungck [1] investigate the idea of compatible mappings which is an active research field to study the common fixed point theorems in compatible mappings. Certain contractive conditions had been satisfies while studying existence and uniqueness of coincidence points and common fixed point theorems [8]. Compatible mapping are used to prove existence theorem in common fixed point theory. The interesting field is common fixed point of non-compatible mappings introduced by pant [2] and also introduce point wise R-weakly commutative of mappings. Weakly compatible pairs of mappings has been introduced by Jungck and Rhoades in 1998 [3] which is the class of mappings commute their coincidence points. Some authors have obtained coincidence point & common fixed point results in complete metric spaces and various metric spaces.

The concept of occasionally weakly compatible (OWC) has been introduces by Al-Thagafi and Shahazad in recent years which is more general than the concept of weakly compatible mappings which have coincidence points. Recently Zhang [11] generalizes the concept of common fixed point theorems for some contractive type mappings. In this paper our aim is to introduce some random fixed point theorems which involve occasionally weakly compatible maps in metric space satisfying generalized contractive conditions. Some common fixed point results for mappings have been satisfying integral type contractive conditions, Meir-Keeler contraction of integral type showed by Suzuki [12] and also X. Zhang [11] form a generalized contractive type condition for a mapping & proved common fixed point theorems in metric space.

Before 1986 Banach contraction principle [7] was the major theme to prove existence and uniqueness of a fixed point, but after that kannan [5] says that to prove fixed point theorem for a map satisfying a contractive condition, it does not require continuity at each point. Suhas Patil & U.P.Dolhare [8] proved some common fixed point theorem using weakly commuting mappings. Jungck [1] first generalized the idea of compatible mapping and after that he generalized weakly compatible mappings.

II. COMPATIBLE MAPPINGS IN METRIC SPACE

We know the maps f and g from X to X commute if and only if fg is equal gf. Suhas S. Patil and U.P.Dolhare [8] proved some common fixed point theorems using the concept of weakly commuting mapping but now a day's new concept has been used called compatibility for proved some fixed point theorem.

Corresponding Author: [1]Suhas Shivajirao Patil*
[1]Research Scholar, SRTMU, Nanded (M.S), India.

Any two self-maps S and T of a set X are Compatible if and only if they commute on the set $\{x \in X : S(x) = T(x)\}$ of coincidence points of S and T. In this paper the following notation are to be used, N-Set of all natural numbers, R-set of all real numbers and Sx and respective all in case of *S(x)* when it convenient.

Definition 1.1[8]: Two self-maps S and T are compatible of a metric space (X, δ) if and only if $lim_{k\to\infty} d(STXk, TSXk) = 0$ when $\{X_k\}$ is a sequence such that, $lim_{k\to\infty} SXk = lim_{k\to\infty} TXk$ for some k in X.

It is clear from the above definition that S and T are non-compatible if there exist at least one sequence $\{X_k\}$ such that $lim_{k\to\infty} Sxk = \lim_{k\to\infty} Txk = P$ for some P belongs to X.

Definition 1.2[8]: let S and T be two self-mappings in a metric space (X, δ) are said to be weakly commuting if it satisfies the following condition

$d(STx, TSx) \le d(Sx, Tx)$, for all x belongs to X.

Remark 1.1: Two commuting mappings are weakly commuting but converse is not true.

Remark 1.2: Two Weakly commuting mappings are compatible but converse is not true.

Definition 1.4: Two Self-mapping S and T of a metric space (X, δ) are weakly compatible, i.e $Sk = Tk$ for some $k \in X$ then $STk = TSk$ if they commute at their coincidence points.

Remark 1.3: Two compatible mappings are weakly compatible.

Theorem 1.1: Let (X, δ) be a metric space in which S and T be continuous self-mappings where if S is a proper mapping then S and T are compatible if and only if

$Sk = Tk$ implies $STk=TSk$.

Corollary 1.1: Let (X, δ) be a metric space, two continuous self-maps are compatible if and only if they commute on their set of coincidence points.

Definition 1.3: Two Self-maps S and T of a metric space (X, δ) will be non-compatible if there exist at least one sequence $\{X_k\}$ in X, such that

$lim_{k\to\infty} Sxk = \lim_{k\to\infty} Txk = P$ for some P belongs to X

and $lim_{k\to\infty} d(STxk, TSxk)$ is non existant.

Theorem 1.2 [8]: Let (X, δ) be a metric space in which S and T be weakly compatible self-mappings satisfying following conditions,

1) S and T satisfying the *(E.A)* property
2) $d(Tl,Tm) < \max\{d(Sl,Sm),[d(Tl,Sl)+d(Tm,Sm)]/2,[d(Tm,Sl)+d(Tl,Sm)]/2\} \forall l \ne m \in X$
3) $Tl \subset Sl$

If Sl or Tl is a complete subspace of X then T and S have a unique common fixed point.

Proof: Since S and T satisfy E.A property then there exist a sequence $\{X_k\}$ in X satisfying

$lim_{k\to\infty} Sxk = \lim_{k\to\infty} Txk = P$ for some P belongs to X.

Let Sx is complete then $lim_{k\to\infty} Sxk = St$ for some t belongs to X. Also $lim_{k\to\infty} Txk = St$, Suppose that St not equal to Tt which implies

$$d(Txk,Tt) < \max\ d\{Sxk,St),[d(Txk,Sxk)+(Tt,St)]/2,[d(Tt,St)+d(St,St)]/2\}$$
$$\le d(Tt,St)/2$$

This is a contradiction.

Hence $Tt=St$, since S and T are weakly compatible $STt=TSt$

$\therefore TTt = TSt = STt = SSt$

And we show that Tt is a common fixed point of T and S, let us consider that $TTt \ne Tt$ then

$$d(Tt,TTt) \le \max\{d(St,STt),[d(Tt,St)+d(TTt,STt)]/2,[d(TTt,St)+d(Tt,STt)]/2\}$$
$$\le \max\{d(Tt,TTt),d(TTt,Tt)\} = d(Tt,TTt)]$$

This is a contradiction,

$$\therefore TTt = Tt \ and\ STt = TTt = Tt.$$

Hence proved.

Corollary 1.2: Let (X, δ) be a metric space in which S and T are continuous self-mapping and S is proper then *Sk= Tk* implies *k=Sk* then *S* and *k* are Compatible.

III, MAIN RESULT

FIXED POINT THEOREM FOR WEAKLY COMPATIBLE MAPPINGS

Corollary 2.1: Let (X, δ) be a metric space in which S and T are two weakly compatible self-mappings such that T and S are satisfy E.A. property T(x) subset of S(x) and

$$F(d(S_l,T_m)) \leq F(M(l,m))) \forall l,m \in X$$

Where A=D if D is infinity and A>D if D is less than infinity,

So, $$M(l,m) = \max\{d(S_l,T_m),[d(T_l,S_l)+d(T_m,S_m)]/2,[d(T_m,S_l)+d(T_l,S_m)]/2\}$$

If *Sx* or *Tx* be a complete space of X than S and have a unique common fixed point.

Corollary 2.2: Let (X, δ) be a metric space in which S and T are two non-compatible weakly compatible self-mappings satisfying following conditions such that

i. $d(T_l,T_m) < \max\{d(S_l,S_m),[d(T_l,S_l)+d(T_m,S_m)]/2,[d(T_m,S_l)+d(T_l,S_m)]/2\}$

ii. $T_x \subset S_x$

Then S and T have a unique common fixed point if S_x and T_x is a complete subspace of X.

Corollary 2.3 Let (X, δ) be a metric space and T be self-mapping then suppose that there exist a map $\beta : X \to R^+$

i. $d(l,T_l) \leq \beta(l) - \beta(T_l) \forall l \in X$

ii. $d(T_l,T_m) < \max\{d(l,m),[d(l,T_m)+d(m,T_l)]/2\} \forall l \neq m \in X$

Then T has a unique fixed point.

Definition 2.1: A pair of mapping S and T are said to be weakly compatible if they commute at coincidence points.

Example 2.1: let the set X = [2,20] in a metric space (X, δ) and define mappings $Q,T : X \to X$ such that *Qx = x if x = 2 (or x > 5), Bx = 6* if *2 < x ≤ 5, Tx = x* if *x = 2* , *Tx = 12* if *2 < x ≤ 5* , *Tx = x-3 if x > 5,* the mapping Q and S are non-compatible since the sequence $\{X_k\}$ in X defined as $X_k = 5 + \frac{1}{n}, n \geq 1$ then $TX_k \to 2, QX_k = 2, TQX_k = 2$ and $QTX_k = 6$. But they are weakly compatible since they commute at coincidence point at $x = 2$.

Remark 2.1: Weakly compatible mappings are need not be compatible.
Let P,Q,S and T are mapping in a metric space (X, δ) into itself satisfying the following conditions,

i. $P(x) \subset T(x)$ and $Q(x) \subset S(x)$ (I)

ii. $\delta(P_x,Q_x) \leq \beta[d(S_x,T_y),d(P_x,S_x),d(Q_{y,}T_y),d(P_x,T_y),d(Q_{y,}S_x)]$ (II)

For all $x, y \in X$ and β be the family of all the mappings.

For any point t_0 in *X* then by (I) we take a point t_1 such that $Tt_1 = Pt_0$ and for this point *t* there exist a point t_2 in *X* such that $St_2 = Qt_1$ and so on.

Theorem 2.1: Let (P,S) and (Q,T) be a pair of weakly compatible self-Mappings of complete metric space (X, δ) satisfying condition (I) and (II) then P, Q, S and T have a unique common fixed point in X.

Proof: Let $\{Y_k\}$ be a Cauchy sequence in X and since X is complete then there exist a point l in S such that $lim_{k\to\infty} Y_k = l$, $lim_{k\to\infty} PX_{2k} = \lim_{k\to\infty} TX_{2k+1} = l$ and $lim_{k\to\infty} QX_{2k+1} = \lim_{k\to\infty} SX_{2k+2} = l$.

$\therefore \quad lim_{k\to\infty} PX_{2k} = \lim_{k\to\infty} TX_{2k+1} = \lim_{k\to\infty} QX_{2k+1} = \lim_{k\to\infty} SX_{2k+2} = l.$

Since $Q(x) \subset S(x)$ there exist a point *u* belongs to *X* such that $l = S_{u..}$

By using (II)

$$\delta(P_x, l) \le \delta(Pu, Qx_{2k-1}) + \delta(Qx_{2k-1}, l)$$
$$\le \beta[\delta(Su, Tx_{2k-1}), \delta(Pu, Su), \delta(Qx_{2k-1}, Tx_{2k-1}), \delta(Pu, Tx_{2k-1}), \delta(Qx_{2k-1}, Su)]$$

Consider the limit as $k \to \infty$ we have,

$$\delta(P_u, l) \le \beta[0, \delta(Pu, Su), 0, \delta(Pu, l), \delta(l, Su)]$$
$$= \beta[0, \delta(Pu, l), 0, \delta(Pu, l), 0)]$$
$$\le \alpha\delta(Pu, l)$$

Where α < 1, we get $l = Pu = Su$.

Since $P(x) \subset T(x)$ then there exist a point *v* belongs to *X* such that $t = T_v$ then again using (II)

$$\delta(t, Bv) = \delta(Pu, Qv) \le \beta(\delta(Su, Tv), \delta(Pu, Su), \delta(Qv, Tv), \delta(Pu, Tv), \delta(Qv, Su))$$
$$= \beta(0, 0, \delta(Bv, l), 0, \delta(Bu, l)) \le \beta(g, g, g, g, g) < g$$

Where $\beta = \delta(l, Bv)$

$$\therefore\ l = Qv = Tv$$

Thus, $Pu = Su = Bu = Tu = l$.

Let the pair of maps P and S are weakly compatible then $PSu = SPu$ Hence $pl = sl$.

Here we show that l is a fixed point of P, If $pl \ne l$ then by (II)

$$\delta(Pl, l) = \delta(Pl, Qv) \le \beta(\delta(Sl, Tv), \delta(Pl, Sl), \delta(Bv, Tv), \delta(Pl, Tv), \delta(Qv, Sl))$$
$$= \beta(\delta(Pl, l), 0, 0, \delta(Pl, l), \delta(Pl, l))$$
$$\le \beta(g, g, g, g, g) < g$$

$\therefore\ pl = l$, hence $pl = Sl = l$.

Similarly, the mapping Q and T are weakly compatible.

We have, $Ql = Tl = l$.

Since,

$$\delta(l, Ql) = \delta(Pl, Qv) \le \beta(\delta(Sl, Tl), \delta(Pl, Sl), \delta(Qv, Tl), \delta(Pl, Tl), \delta(Ql, Sl))$$
$$= \beta(\delta(l, Tl), 0, 0, \delta(l, Tl), \delta(l, Tl))$$
$$\le \beta(g, g, g, g, g) < g, \quad \text{where } g = \delta(l, Tl) = \delta(l, Ql)$$

Hence, $l = Pl = Ql = Sl = Tl$ and l is a common fixed point of P, Q, S and T.

Hence { Y_k } is Cauchy sequence.

This completes the proof of theorem.

IV. CONCLUSION

In this paper we see the generalization of the result by G. Jungck [6] by using weakly compatible mapping without continuity at *S* and *T*.

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