Total No. of Printed Pages: 3

SUBJECT CODE NO:- B-2040
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. F. Y. (Sem-I)
Examination November/December- 2022
Mathematics MAT - 102
(Differential Equations)

[Time: 1:30 Hours] **[Max. Marks:50]**

"Please check whether you have got the right question paper."

N.B.
1) Attempt all questions.
2) Figures to the right indicates full marks.

Q.1 A) Attempt any one. 08

a) Explain the method of Solving differential equation $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x or constants.

b) Explain the method of solving differential equation

$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1}y}{dx^{n-1}} + P_2 \frac{d^{n-2}y}{dx^{n-2}} + \cdots + P_n y = X$,

Where $P_1, P_2, \ldots, P_n$ are constants and X is a function of x.

B) Attempt any one. 07

c) Solve the simultaneous equations

$\frac{dx}{dt} - 7x + y = 0 ; \frac{dy}{dt} - 2x - 5y = 0$

d) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 2e^{2x}$

Q.2 A) Attempt any one. 08

a) Explain the method of solving the differential equation

$x^n \frac{d^n y}{dx^n} + P_1 x^{n-1} \frac{d^{n-1}y}{dx^{n-1}} + \cdots + P_{n-1} x \frac{dy}{dx} + P_n y = X$,

Where $P_1, P_2, P_3 \ldots, P_n$ are constants and X is a function of x.

b) Solve $x^2 \frac{d^2y}{dx^2} + 7x \cdot \frac{dy}{dx} + 5y = x^5$

B) Attempt any one. 07

c) Solve $\frac{d^2y}{dx^2} - 4y = 2 \cdot \sin\left(\frac{1}{2} \cdot x\right)$

d) Solve $(5 + 2x)\frac{d^2y}{dx^2} - 6(5 + 2x)\frac{dy}{dx} + 8y = 0$

Q.3 A) Attempt any one. 05

a) Explain the method of solving equation $\frac{d^n y}{dx^n} = f(x)$

b) Derive the Partial differential equation by the elimination of the arbitrary constants from the equation $\phi(x, y, z, a, b) = 0$.

B) Attempt any one. 05

c) Solve $\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx}$

d) Form a Partial differential equation by eliminating the arbitrary function from $z = F(x^2 + y^2)$.

Q.4 Choose correct alternative. 10

i) The integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$ is ______.

(a) $e^{\int Pdx}$

(b) $e^{-\int Pdx}$

(c) e^x

(d) e^{Px}

ii) The general solution of the differential equation $\frac{d^2y}{dx^2} - a^2y = 0$ is ______.

(a) $y = (c_1 + c_2x)e^{ax}$

(b) $y = (c_1 + c_2x)e^{-ax}$

(c) $y = c_1e^{ax} + c_2e^{-ax}$

(d) None of these

iii) The particular integral of the differential equation $\frac{d^2y}{dx^2} - y = 2 + 5x$ is ______.

(a) $2 + 5x$

(b) $-2 - 5x$

(c) $-2 + 5x$

(d) $2 - 5x$

iv) The Solution of the Simultaneous equation $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$ is ______.

(a) $x = c_1 y$ and $x = c_2 z$

(b) $x = c_1 y^2$ and $x = c_2 z^2$

(c) $x = c_1 x^2$ and $x = c_2 z^2$

(d) None of the above

v) The Partial differential equation corresponding to the equation $z = (x + a)(y + b)$ is ______.

(a) $z = p^2 q^2$

(b) $z = p + q$

(c) $z = p - q$

(d) $z = pq$

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SUBJECT CODE NO:- B-2039
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. F.Y. (Sem-I)
Examination November/December- 2022
Mathematics MAT – 101
Differential Calculus

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
1) Attempt all question
2) Figure to the right indicate full marks.

Q. 1 (A) Attempt any one: 08

(a) If U and V be two functions of x possessing derivatives of the nth order then prove that, $(UV)_n = U_n + nC_1 U_{n-1} V_1 + nC_2 U_{n-2} V_2 +$ ________ + ________ + $nC_r U_{n-r} V_r +$ __________ + $^nC_n UV_n$

(b) Show that, if fis finitely derivable at c, then f is also continuous at c.

(B) Attempt any one: 07

(c) If $f(x)=x^2 \sin(1/x)$ when x≠o and f(o), show that f is derivable for every value of x but the derivative is not continuous for x=o

(d) If y= a cos (log x)+b sin (log x). show that,

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

Q. 2 (A) Attempt any one: 08

(a) If a function f is,

i. Continuous in closed interval [a,b]
ii. Derivable in the open interval (a,b)
iii. f(a)=f(b), Then, Prove that, three exists at least one value c∈ (a, b) such that, $f^1(c)=0$

(b) If $z=f(x_1y)$ is homogeneous function of x,y of degree n, then prove that,

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\,\partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z$$

(B) Attempt any one:

(c) Discuss applicability of Rolle's theorem to the function $f(x) = |x|\ in\ [-1,1]$ 07

(d) If z=(x+y) ∅ ($\frac{y}{x}$), where ∅ is any x $\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$

Q. 3 (A) Attempt any one: 05

(a) Prove that, the gradient of scalar point function is a vector point function

(b) Prove that, grad f(r) x $\underset{r}{\rightarrow}$= 0 where, $r = \sqrt{x^2 + y^2 + z^2}$ and $\underset{r}{\rightarrow} = x\bar{\imath} + y\bar{\imath} + z\bar{k}$

(B) Attempt any one: 05

(c) Show that,

Grad $(\underset{f}{\rightarrow}.\underset{g}{\rightarrow}) = \underset{f}{\rightarrow}\times\ curl\ \underset{g}{\rightarrow} + \underset{g}{\rightarrow}\times\ curl\ \underset{f}{\rightarrow} + \left(\underset{f}{\rightarrow}\ \nabla\right)\underset{g}{\rightarrow} + (\underset{g}{\rightarrow}.\nabla)\ \underset{f}{\rightarrow}$

(d) Show that $\forall\ x\ \in R$

Sin x$=x - \frac{x^3}{3!} + \frac{x^5}{5!} +$ ________

Q. 4 Choose the correct alternative. 10

i. $\lim_{x\to o} \sin(\frac{1}{x})$ ________

a) Exists
b) Is equal to zero
c) Is equal to ∞
d) Does not exists

ii. If $x^p\ y^q = (x+y)^{p+q}$ Then, $\frac{dy}{dx}$ is equal to________

a) $\frac{y}{x}$
b) $\frac{py}{qx}$
c) $\frac{x}{y}$
d) $\frac{qy}{px}$

iii. If x=t-sin t, y=1-cost, Then $\frac{d^2y}{dx^2}$ at $(\pi, 2)$ will be_______

a) 0

b) 1

c) π

d) ∞

iv. If f is continuous in [a,b] and differentiable in (a,b) then three exists at least one point C in (a,b) such that $f^1(c)$ is equal to_________

a) $\frac{f(b)+f(a)}{b+a}$

b) $\frac{f(b)-f(a)}{b+a}$

c) $\frac{f(b)-f(a)}{b-a}$

d) $\frac{f(b)+f(a)}{b-a}$

v. curl $\underset{r}{\rightarrow}$=______

a) 1

b) 2

c) 3

d) 0

Total No. of Printed Pages:3

SUBJECT CODE NO:- B-2054
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. F. Y. (Sem-II)
Examination November/December- 2022
Mathematics MAT - 201
(Integral Calculus)

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
i) Attempt all the Questions.
ii) Figures to the right indicate full marks.

Q.1 A) Attempt any one. 08

a) Obtain a reduction formula for $\int x^m (\log x)^n dx$ and evaluate $\int_0^1 x^4 (\log x)^3 dx$.

b) Obtain reduction formula for $\int \cos^n x \, dx$. Where n is positive integer. Also find $\int_0^{\pi/2} \cos^8 x \, dx$.

B) Attempt any one. 07

c) Evaluate $\int_2^3 \frac{(x^2+1)}{(2x+1)(x^2-1)} dx$.

d) Evaluate $\int \frac{dx}{1-x^6}$

Q.2 A) Attempt any one 08

a) Evaluate $\int_a^b \sin hx \, dx$ as the limit of sum.

b) Find the area between the curve

$$x(x^2+y^2) = a(x^2-y^2)$$

and its asymptote. Also find the area of its loop.

B) Attempt any one 07

c) Find the perimeter of the loop of the curve

$$9\,ay^2 = (x-2a)(x-5a)^3$$

d) Find the volume of the solid obtained by revolving the cardioide

$r = (1 + \cos\theta)$ about the initial line.

Q.3 A) Attempt any one 05

a) Show that

$$\frac{1}{3}\int_s \vec{r}\,.\,d\vec{a} = V$$

Where V is the volume enclosed by the surface S.

b) Verify stoke's theorem for the function $\vec{F} = x(\vec{\imath}x + \vec{\jmath}y)$, integrated round the square in the plane $z = 0$ whose sides are along the line.

$x = 0, y = 0, x = a, y = a$

B) Attempt any one 05

c) Evaluate $\int_c \vec{F}.\,d\vec{r}$ where

$\vec{F} = (2y + 3)\vec{\imath} + xz\vec{\jmath} + (yz - x)\vec{k}$

Along the path C is straight line joining (0,0,0) to (2,1,1)

d) Evaluate $\int_s \frac{\vec{r}}{r^3}\,d\vec{a}$

Where S denotes the sphere of radius a with center at the origin.

Q.4 Choose the correct alternatives. 10

1) $\int \frac{dx}{3x-4} =$ ____

a) $3\log(3x - 4)$ b) ${}^1/_3 \log(3x)$

c) ${}^1/_3 \log(3x - 4)$ d) ${}^1/_4 \log(3x - 4)$

2) $\int^{\pi/2} \sin^8 x\, dx$

a) $\frac{35\pi}{256}$ b) $\frac{256\pi}{35}$ c) $\frac{35}{256\pi}$ d) $\frac{256}{35\pi}$

3) The perimeter of the curve $r = 2\cos\theta$ is _____.

a) $\frac{\pi}{2}$ b) π c) $\frac{3\pi}{2}$ d) 2π

4) The volume generated by revolving about the x-axis an area bounded by the curve $y = t(x)$ and the two ordinates $x = a$ and $y = b$ is given by

a) $\int_a^b y^2 dx$ b) $1/2 \int_a^b y^2 dx$

c) $\frac{\pi}{2}\int_a^b y^2 dx$ d) $\pi \int_a^b y^2 dx$

5) Value of $\int(xdy - ydx)$ around the circle $x^2 + y^2 = 1$ is

a) 0 b) $\frac{\pi}{2}$ c) π d) 2π

Total No. of Printed Pages: 03

SUBJECT CODE NO:- B-2055
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. F.Y (Sem-II)
Examination November/December- 2022
Mathematics MAT - 202
(Geometry)

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B i) Attempt all questions.
ii) figures to the right indicate full marks.

Q. 1 A) Attempt any one: 08

a) Show that the equation of the first degree in x, y, z represents a plane.

b) Find the angle between the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and the plane $ax + by + cz + d = 0$.

B) Attempt any one: 07

c) Find the equation of the plane passing through the lines of intersection of the planes $2x - y = 0$ and $3z - y = 0$ are perpendicular to the plane $4x + 5y - 3z = 8$.

d) Find the equation of the plane containing the line $2x - 5y + 2z = 6, 2x + 3y - z = 5$ and parallel to the line $x = \frac{-y}{6} = \frac{z}{7}$.

Q. 2 A) Attempt any one: 08

a) Find the condition that two straight lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}, \frac{x-z_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ are coplanar.

b) Define a sphere, obtain the equation to a sphere on line joining the point $(x_{1,}y_1, z_1)$ and $(x_{2,}y_2, z_2)$ as a diameter.

B) Attempt any one: 07

a) Find the equation of the line through the point (1,2,3) parallel to the line $x - y + 2z = 5, 3x + y + z = 6$.

b) Show that the two spheres

$x^2 + y^2 + z^2 - y + 2z = 0\ , x - y + z - 2 = 0\ ;$

$x^2 + y^2 + z^2 + x - 3y + z - 5 = 0, 2x - y + 4z - 1 = 0$

line on the same sphere and find its equation.

Q. 3 A) Attempt any one 05

a) Prove that every section of a right circular cone by a plane perpendicular to its axis is a circle.

b) Fine the condition that the plane lx+my+nz=p, should touch the central conicoid $ax^2 + by^2 + cz^2 = 1.$

B) Attempt any one 05

c) Find the length of the perpendicular from the point P (5,4, -1) upon the line $\frac{1}{2}(x-1) = \frac{1}{9}y = \frac{1}{5}z.$

d) Fine the equation to the right circular cone whose vertex is at origin, the axis along x-axis and semi-vertical angle is α .

Q.4 Choose the correct alternative: 10

1) The angle between the two planes $3z - 4y + 5z = 0\ and\ 2x - y - 2z = 5\ is$________

a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{4}$

2) The foot of perpendicular form (2,3,4) to the plane $x + y - z + 4 = 0$ is _______

a) $\left(\frac{-1}{3}, \frac{4}{3}, \frac{17}{3}\right)$ b) $(1/3, -4/3, \frac{17}{3})$ c) $\left(\frac{1}{3}, \frac{4}{3}, \frac{17}{3}\right)$ d) $(1/3,\ 4/3, -17/3)$

3) Centre of the sphere $x^2 + y^2 + z^2 - 4x + 6y - 8z + 8 = 0$ is ________

a) (2,-3,4) b) (2,3,4) c) (-2,-3,-4) d) (1,2,3)

4) The plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ at the point ____________.

a) (1,-4,-2) b) (-1,4,-2) c) (-1,4,2) d) (1,4,-2).

5) The straight line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ is parallel to the plane $ax + by + cz + d = 0$, if __

a) $al + bm + cn = 1$

c) $al + bm + cn = 0$

b) $ax_1 + by_1 + cz_1 = 1$

d) $ax_1 + by_1 + cz_1 = 0$

Total No. of Printed Pages: 03

SUBJECT CODE NO:-B- 2117
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. S. Y. (Sem-III)
Examination November/December- 2022
Mathematics MAT - 303
Mechanics-I

[Time: 1:30 Hours] **[Max. Marks:50]**

N.B "Please check whether you have got the right question paper"

i) All questions are compulsory.
ii) Figures to the right indicate full marks.
iii) Draw well-labelled diagrams whenever necessary.

Q.1 (a) Attempt any one of the following: [08]

i) State and prove Lami's theorem

ii) Show that C divides the line joining the points of application of two like parallel forces internally in the inverse ratio of their magnitudes.

(b) Attempt any one of the following: [07]

i) The forces of magnitudes 2, 3, 4, 5 and 6 kg are acting on one of the angular points of rectangular hexagon towards the other five angular points taken in order. Find the magnitude and direction of the resultant force.

ii) Three forces of the magnitudes P, Q, R acting on a particle are in equilibrium and the angle between P and Q is double the angle between P and R. Show that $R^2 = Q(Q - P)$

Q.2 (a) Attempt any one of the following: [08]

i. Prove that the necessary and sufficient condition that a given system of forces acting upon a rigid body is in equilibrium is that the force force-sum and moment- sum must separately vanish.

ii. Prove that the sum of the vector moments of two like parallel force acting on a rigid body about any point equals to the vector moment of their resultant about the same point.

(b) Attempt any one of the following: [07]

i. A force $\vec{F}$ of magnitude 8 units acts at a point P(2, 3, 4) along the line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ Find the vector moment of the force $\vec{F}$ about y-axis.

ii. A uniform string is bend into the form of a ΔABC with sides a, b, c. Show that the distances of the C.G. of the ΔABC from their sides BC, CA and AB respectively are in the ratio $\frac{b+c}{a}:\frac{c+a}{b}=\frac{a+b}{c}$

Q.3 (a) Attempt any one of the following: [05]

i. Prove that the C.G. of the uniform parallelogram is at the point of intersection of the diagonals of the parallelogram.

ii. A system of the forces acting upon a rigid body is equivalent to a force at any arbitrary point together with a couple.

(b) Attempt any one of the following: [05]

i. Find the vector moment of a force $\vec{F} = \vec{\imath} + 2\vec{\jmath} + 3\vec{k}$acting at a point (-1, 2, 3) about origin.

ii. Two forces of magnitudes $(P + Q)$ and $(P - Q)$ make an angle 2θ with each other and their resultant force makes an angle α with the bisector of the angle between them. Prove that

$$\frac{P}{Q} = \frac{tan\theta}{\tan\alpha}$$

Q.4 Choose the correct alternative and rewrite the sentence: [10]

(a) If two forces $\vec{P}$ and $\vec{Q}$ acting at an angle θthen the magnitude R of their resultant force is given by -------

i. $R = \sqrt{P^2 + Q^2 - 2PQ\,cos\theta}$
ii. $R = \sqrt{P^2 + Q^2 + 2PQ\,cos\theta}$
iii. $R = \sqrt{P^2 + Q^2 + 2PQ\,sin\theta}$
iv. $R = \sqrt{P^2 + Q^2 - 2PQ\,sin\theta}$

(b) The direction of the resultant of the unlike parallel forces is the same as that of the -----------

i. smaller component
ii. both components
iii. opposite to the smaller component
iv. bigger component

(c) If any number of forces acting on a particle be represented in magnitude and direction by the sides of a polygon taken in order, then the forces are in --------------

i. equal
ii. same direction
iii. equilibrium
iv. opposite direction

(d) If the three forces acting on a particle be represented in magnitude and direction by the three sides of a triangle, taken in order, then ------------

i. the forces coincide each other
ii. the forces are in equilibrium
iii. the forces are non-coplanar
iv. the forces are not in equilibrium

(e) Centroid of the weighted point --------------

i. does not exists
ii. exists but is not unique
iii. exists and is unique
iv. does not exists but is unique

Total No. of Printed Pages: 2

SUBJECT CODE NO:- B-2050
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. S.Y. (Sem-III)
Examination November/December- 2022
Mathematics MAT – 301
Number Theory

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
i) All questions are compulsory.
ii) Figures to the right indicate full marks.

Q.1 a) Attempt any one of the following: 08

i. If k> 0, then prove that gcd (ka, kb) = k gcd(a, b).

ii. For integers a, b, c, prove the following

α) if $a|b$ and $b|c$ then $a|c$,

β) if $a|b$ and $a|c$ then $a|(bx + cy)$ for arbitrary integers x and y

b) Attempt any one of the following: 07

i. If a is odd integer, then prove that $32|(a^2 + 3)(a^2 + 7)$.

ii. Find all solutions in the integers of the Diophantine equation $24x + 138y = 18$.

Q.2 a) Attempt any one of the following: 08

i. State and prove Chinese remainder theorem.

ii. If $p$is prime number, then prove that (p-1)!= -1(modp).

b) Attempt any one of the following: 07

i. Solve the linear congruence 25x = 15(mod29).

ii. If gcd (a, 133) = gcd(b, 133) = 1, then show that $133 |a^{18} - b^{18}$.

Q.3 a) Attempt any one of the following: 05

i. If p is a prime number and $p|ab$, then prove that $p|a$ or $p|b$.

ii. If F is multiplicative function and is defined by

$$F(n) = \sum_{d|n} f(d),$$

then prove that f is multiplicative function.

b) Attempt any one of the following: 05

i. Calculate $\phi(360)$.

ii. Find the values of τ(180) and $\sigma(180)$.

Q.4 Choose the correct alternative and rewrite the sentence: 10

1) $\gcd(-12,30) = - - - - - - -$

a) 6 b) 4 c) 3 d) 1

2) The number of solutions of linear congruence $6x \equiv 15 (\text{mod} 21)$ is ..

a) 6 b) 3 c) 1 d) 15

3) The value of $\mu(10)$ is -------------

a) -1 b) 0 c) 5 d) 1

4) If $\gcd(a,b) = d$, then $\gcd\left(\frac{a}{d}, \frac{b}{d}\right)$ = --------

a) 1 b) d c) $\frac{1}{d}$ d) ab

5) If $a|bc$ with $\gcd(a,b) = 1$ then ------------

a) $b|a$ b) $a|c$ c) $c|a$ d) $a = b$

Total No. of Printed Pages: 2

SUBJECT CODE NO:- B-2051
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. S.Y. (Sem-III)
Examination November/December- 2022
Mathematics MAT - 302
Integral Transforms

[Time: 1:30 Hours] **[Max. Marks:50]**

Please check whether you have got the right question paper.

N.B All questions are compulsory ,between internal choice in available
Figures to the right indicate full marks

Q.1 (a) Attempt any one of the following: 08

i. If $L^{-1}\{$f(s)$\}$ = F(t), then prove that $(L^{-1})f^{n}$(s)$\}$ = $(-1)^{n}t^{n}F(t)$

ii. Derive the relation between Fourier transform and Laplace transform.

(b) Attempt any one of the following: 07

i. Using Laplace transform, find the solution of the differential equation $(D^2 + D)y = t^2 + 2t,$ where y(0)=4 $y'(0) = -2$

iii. Find the value of $L^{-1}\left\{\frac{1}{s(s+1)^3}\right\}$

Q.2 a) Attempt any one of the following 08

i. If L{F(t)}=f(s),then prove that $\lim_{s\to\infty} F(t) = .\lim_{t\to 0} sf(s)$

ii. If $\tilde{f}\ (s)$and $\tilde{g}$ (s) are Fourier transforms of f(x)and g(x) respectively, then prove that

$F\{a\ f(x) + bg(x)\}$ = a $\tilde{f}\ (s) + b\ \tilde{g}$ (s)

Where a and b are constants

b) Attempt any one of the following 07

i. Prove that $L^{-1}\left\{\tan^{-1}\frac{2}{s^2}\right\} = \frac{2}{t}\ sintsinh\, t.$

ii. Using Laplace transform, prove that $\int_0^\infty te^{-3t}sintdt = \frac{3}{50}$

Q.3 (a) Attempt any one of the 05

i. If L{F(t)} = f(s), then prove that L{e^{at}F(t)} = f(s + a).

ii. If f(s) is the Fourier transform F(x), then prove that the Fourier transform of F'(x) is equal to is f(s).

(b) Attempt any one of the following: 05

i. Evaluate the integral

$$\int_0^{\infty} e^{ax} x^{m-1} sinbxdx$$

ii. Evaluate L{sin at - at cos at}.

Q.4 Choose the correct alternative and rewrite the sentence: 10

(a) If $\int_0^{\infty} e^{-} dx = \frac{\sqrt{\pi}}{2}, then \int_{-\infty}^{\infty} e^{-x^2 dx} =$_______

i. $\frac{\sqrt{\pi}}{2}$

ii. $\sqrt{\frac{\pi}{2}}$

iii. $\sqrt{\pi}$

iv. 0

(b) $L\{2t^3 - 6t + 8\} =$_______

i. $\frac{12}{s^3} - \frac{6}{s^2} + \frac{8}{s}, s > 0$

ii. $\frac{6}{s^4} - \frac{6}{s^2} + \frac{8}{s}, s > 0$

iii. $\frac{12}{s^4} - \frac{6}{s^2} + \frac{8}{s}, s > 0$

iv. $\frac{12}{s^4} - \frac{6}{s^2} + \frac{8}{s}, s > 0$

(c) $L^{-1}\left\{\frac{1}{s-a}\right\} =$____$, s > a$

i. ae^t

ii. a^{at}

iii. a^{-at}

iv. ae^{-t}

(d) The sine transform of f(x)=$\frac{1}{x}$ is____________

i. $\sqrt{\pi}$

ii. π

iii. 2π

iv. $\frac{\pi}{2}$

(e) $L\{\sinh at\} =$______

i. $\frac{a}{s^2-a^2}$

ii. $\frac{s}{s^2-a^2}$

iii. $\frac{a}{s^2+a^2}$

iv. $\frac{s}{s^2+a^2}$

Total No. of Printed Pages:2

SUBJECT CODE NO:- B-2065
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. S.Y. (Sem-IV)
Examination November/December- 2022
Mathematics MAT - 401
Numerical Methods

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
i) Attempt all questions.
ii) Figure to the right indicate full marks.
iii) Use of non-programmable calculator and logarithmic table is allowed.

Q.1 A) Attempt any one: 08

a) Derive newton – Raphson formula for finding real roots of an equation f(x) = 0.

b) Derive Newton's general interpolation formula.

B) Attempt any one:

c) Obtain a root, correct to four decimal places, which lies between 2 and 3 of the equation 07
$f(x) = x^3 - 2x - 5 = 0$,
by Using the method of false position.

d) Certain corresponding values of x and $\log_{10}^{x}$ are (300, 2.4771), (304, 2.4829), (305; 2.4843) and (307, 2.4871) Find $\log_{10} 301$.

Q.2 A) Attempt any one: 08

a) Define chebyshev polynomial and prove the recurrence relation
$T_{n+1}(x) = 2x\,T_n(x) - T_{n-1}(x)$.
Where $T_n(x)$ is a chebyshev polynomial of degree n.

b) Explain the Gaussian elimination method for solving system of linear equation.

B) Attempt any one: 07

c) Fit a straight line of the form $Y = a_0 + a_1 x$ to the data.

x	1	2	3	4	6	8
y	2.4	3.1	3.5	4.2	5.0	6.0

d) Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$$

Q.3 A) Attempt any one: 05

a) Explain Picard's method of successive approximations to solve the differential equation $y' = f(x,y)$ With the initial condition $y(x_0) - y_0$

b) Prove that the Newton-Raphson method has quadratic convergence.

B) Attempt any one: 05

c) Using Euler's method, solve the differential equation $\frac{dy}{dx} + 2y = 0, \quad y(0) = 1$ take h=0.1 and obtain y(0.1), y(0.2) and y(0.3).

d) Using the method of separation of symbols, show that

$$\Delta^n u_{x-n} = u_x - n u_{x-1} + \frac{n(n-1)}{2} u_{x-2} + \cdots + (-1)^n u_{x-n}$$

Q.4 Choose the correct alternative. 10

i) Rate of convergence of Newton-Raphson method is _________
a) Linear b) Quadratic c) Cubic d) Biquadratic

ii) $\Delta^2 y_1 = ---$
a) $y_2-2y_1+y_0$ b) $y_3+2y_2+y_1$ c) $y_3-y_2+y_1$ d) $y_3-2y_2+y_1$

iii) The chebyshev polynomial of degree one is ______
a) x b) $2x^2-1$ c) $2x^2+1$ d) 1

iv) The eigenvalues of the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ are ________
a) 3,2 b) -3,-2 c) 1,-1 d) 0,4

v) Newton's forward difference interpolation formula is applicable only when the arguments are ________
a) Equally spaced b) Unequally spaced
c)Both equally and unequally spaced d) None of these

Total No. of Printed Pages:2

SUBJECT CODE NO:- B-2066
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. S.Y (Sem-IV)
Examination November/December- 2022
Mathematics MAT - 402
Partial Differential Equation

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
1) All questions are compulsory
2) Figures to the right indicate full marks

Q.1 A) Attempt any one 08

i) Explain the method of obtaining complementary function of $\left(A_o D^n + A_1 D^{n-1}D' + -- + A_n D'^n\right)z = f(x,y)$

ii) Explain the method of obtaining complete general integral of $f_1(x,p) = f_2(y,q)$

B) Attempt any one: 07

iii) Solve: $x^2p + y^2q = z^2$

iv) Solve :$pz = 1 + q^2$

Q.2 A) Attempt any one: 08

a) Explain Jacobi's method to solve $f(x_1, x_2, x_3, p_1, p_2, p_3) = 0$

b) Discuss Monge's method to solve $Rr + Ss + Tt = V$ where R,S,T and V are functions of x,y,z,p and q

B) Attempt any one 07

c) Solve $(p^2 + q^2)y = qz\, by$ using charpit's method

d) Solve: r+5s +6t=0

Q.3 A) Attempt any one 05

a) With usual notations prove that $\frac{1}{F(D^2, DD', D'^2)}\cos(ax + by) = \frac{\cos(ax+by)}{F(-a^2, -ab, -b^2)}$; if $F(-a^2, -ab, -b^2) \pm 0$

b) Find the general solution of $(D - mD' - k)z = 0$

B) Attempt any one 05

c) Solve : $\frac{\partial^2 Z}{\partial x \partial y} = \frac{1}{xy}$

d) Solve : $(D^2 - 2DD' + D'^2)Z = e^{x+2y}$

Q.4 Choose the correct alternatives 10

1) The Lagrange's auxiliary equation of $P_1 \frac{\partial z}{\partial x_1} + P_2 \frac{\partial z}{\partial x_2} + --- + P_n \frac{\partial z}{\partial x_n} = R$ are ------

a) $\frac{dx_1}{1} = \frac{dx_2}{1} = ----- = \frac{dx_n}{1}$

b) $\frac{dx_1}{P_1} = \frac{dx_2}{P_2} = ------ = \frac{dx_n}{P_n}$

c) $P_1\, dx_1 = P_2 dx_2 = -----= P_n dx_n$

d) None of these

2) The complete integral of $z = px + qy + pq$ is ------

a) $z = ax + by$ b) $z = ax + ab$ c) $z = ax + by + ab$ d) $z = a + b$

3) The complementary function of $(D^2 - 2DD' + D'^2))$ z=sin (2x+3y) is ---------

a) $z = \phi_1\,(y + x) + x\phi_2(y + x)$

b) $z = \phi_1(y+x) + \phi_2(y + x)$

c) $z = \phi_1(y - x) + \phi_2(y - x)$

d) $z = \phi_1(y - x) + x^2\phi_2(y - x)$

4) The value of $\frac{1}{F(D,D')}\, e^{ax+by}$ = -----

a) $\frac{1}{F(a,b)}\, e^{ax}, if\ F(a,b) \neq 0$

b) $\frac{1}{F(a,b)}\, e^{by}, if\ F(a,b) = 0$

c) $\frac{1}{F(a,b)}\, e^{ax+by}, if\ F(a,b) = 0$

d) $\frac{1}{F(a,b)}\, e^{ax+by}, if\ F(a,b) \neq 0$

5) The direction ratios of the normal at a point (x, y, z) to the surface given by $Pp + Qq = R$ are ---

a) p,q,1 b) p,q,-1 c) 1,1,1 d) P,Q,R

Total No. of Printed Pages: 02

SUBJECT CODE NO:- B-2115
FACULTY OF SCIENCE & TECHNOLOGY
B. Sc. T.Y. (Sem-V)
Examination November/December- 2022
Mathematics
Ordinary Differential Equation -I 504

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
1) All questions are compulsory
2) Figures to the right indicate full marks

Q.1 A) Attempt any one :

a) Consider the equation $y' + ay = b(x)$ where a is a constant and b is continuous function on an interval I. If x_o is a point in I and C is any constant. Prove that the function ϕ defined by 08

$$\phi(x) = e^{-ax} \int_{x_o}^{x} e^{at} b(t)dt + ce^{-ax}$$

is a solution of this equation? Also prove that every solution has this form

b) If r is a root of multiplicity m of a polynomial P. deg $P \geq 1$ then prove that 08
$P(r) = P'(r) = --------- = P^{(m-1)}(r) = 0 \; and \; P^{(m)}(r) \neq 0$

B) Attempt any one

c) Find all solutions of the equation $y' + 2xy = xe^{-x^2}$ 07

d) If ϕ be the solution of $y' + iy = x$ such that $\phi(0) = 2$ find $\phi(\pi)$ 07

Q.2 A) Attempt any one

a) Prove that two solutions ϕ_1, ϕ_2 of $L(y) = y'' + a_1y' + a_2y = 0$ are linearly independent on an interval I if and only if , $W(\phi_1, \phi_2)(x) \neq 0$ for all x in I 08

b) If ϕ_1, ϕ_2 are two solutions of $L(y) = y'' + a_1y' + a_2y = 0$ on an interval I containing a point x_o then prove that 08
$W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_o)} W(\phi_1, \phi_2)(x_o)$

B) Attempt any one :

c) Find all solutions of the equation $y'' + 4y = cosx$ 07

d) Find the solution of the following initial value problem 07
$y'' + (4i + 1)y' + y = 0$, $y(0) = 0, y'(0) = 0$

Q.3 A) Attempt any one:

a) Prove that for all real θ 05
$e^{i\theta} = cos\theta + i\ sin\theta$

b) Prove that for any real x_o and constants $\propto, \beta$ there exists a solution ϕ of the initial value problem 05
$L(y) = y'' + a_1 y' + a_2 y = 0$
on $-\infty < x < \infty$

B) Attempt any one

c) Find the two square roots of i . 05

d) Find all solutions ϕ of $y'' + y = 0$ 05
$\phi(0) = 0, \phi'(\pi/2) = 0$

Q.4 Choose the correct alternative 10

1) The wronskian of the functions
$\phi_1(x) = \sin x$, $\phi_2(x) = e^{ix}$ *is*
a) 0 b) 1 c) -1 d) None of these

2) The roots of the equation $Z^2 + Z - 6 = 0$ are
a) -3,2 b) 2,3 c) 3,-2 d) none of these

3) If $\phi(x) = e^{iax}$ where a is a real constant then $\phi''(x) + a^2\phi(x)$ =-------
a) 1 b) 0 c) e^{iax} d) none of these

4) $\phi(x) = e^{-\sin x}$ is a solution of the differential equation?
a) $y' + (cosx)y = 0$ b) $y' - (cosx)y = 0$ c) $y' + (sinx)y = 0$ d) None of these

5) All solutions of $y'' + \omega^2 y = 0$ are of the to form
a) $Ge^{i\omega x} - c_2 e^{-i\omega x}$ b) $Ge^{\omega x} + c_2 e^{-\omega x}$ c) $Ge^{i\omega x} + c_2 e^{-i\omega x}$ d) none of these

Total No. of Printed Pages: 3

SUBJECT CODE NO:- B-2046
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. T. Y. (Sem-V)
Examination November/December- 2022
Mathematics MAT - 501
Real Analysis – I

[Time: 1:30 Hours] **[Max. Marks:50]**

"Please check whether you have got the right question paper."

N.B.
1) All questions are compulsory.
2) Figures to the right indicate Full marks.

Q.1 A] Attempt any one:

a) Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent. 08

b) Define Cauchy sequence. 08
If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ converges then prove that $\{S_n\}_{n=1}^{\infty}$ is a Cauchy sequence.

B] Attempt any one:

c) If $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers diverging to infinity, then prove that 07
$\lim_{n\to\infty} \text{Sup}\, S_n = \infty = \lim_{n\to\infty} \inf S_n$.

d) For $n \in I$, let $S_n = \frac{1\cdot 3\cdot 5\cdot \ldots (2n-1)}{2\cdot 4\cdot 6\cdot \ldots 2n}$ 07
Prove that $\{S_n\}_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} S_n \le \frac{1}{2}$.

Q.2 A] Attempt any one:

a) Let $\sum_{n=1}^{\infty} a_n$ be a series of nonzero real numbers and let $a = \lim_{n\to\infty} \inf \left|\frac{a_{n+1}}{a_n}\right|$, 08
$A = \lim_{n\to\infty} \text{Sup} \left|\frac{a_{n+1}}{a_n}\right|$ Then prove that $\sum_{n=1}^{\infty} |a_n| < \infty$ if $A < 1$.

b) If $\sum_{n=1}^{\infty} a_n$ is a divergent series of positive numbers then prove that there is a sequence $\{\epsilon_n\}_{n=1}^{\infty}$ of positive numbers which converges to zero but For which $\sum_{n=1}^{\infty} \epsilon_n a_n$ still diverges. 08

B] Attempt any one:

a) Does the series 07

i) $\sum_{n=1}^{\infty} \frac{n+1}{n+2}$ and

ii) $\sum_{n=1}^{\infty} \frac{n+1}{10^{10}(n+2)}$ converge or diverge?

Justify your answer.

b) Prove that the series $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{1}{n}$ converges. 07

Q.3 A] Attempt any one:

a) If $u_1, u_2, \dots, u_n$ are implicit functions of $x_1, x_2, \dots, x_n$ then prove that 05

$$\frac{\partial(u_1, u_2, \dots, u_n)}{\partial(x_1, x_2, \dots, x_n)} = (-1)^n \frac{\frac{\partial(F_1, F_2, \dots, F_n)}{\partial(x_1, x_2, \dots, x_n)}}{\frac{\partial(F_1, F_2, \dots, F_n)}{\partial(u_1, u_2, \dots, u_n)}}$$

b) Prove that the inverse image of the intersection of two sets is the intersection of the inverse images. 05

B] Attempt any one :

c) Find the Jacobian of $y_1, y_2, \dots, y_n$ being given $y_1 = 1 - x_1$, $y_2 = x_1(1 - x_2), \dots, y_n = x_1 x_2 \dots x_{n-1}(1 - x_n)$. 05

d) If $x = c \cos u \cos hv$, $y = c \sin u \sin hv$, 05

Prove that $\frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{2} c^2 (\cos 2u - \cos h2u)$.

Q.4 Choose correct alternative of the following. 10

1) If $f: A \rightarrow B$ is a function defined by $f(x) = \sqrt{x}$ then
   a) $A = B = \mathbb{IR}$
   b) $A = \mathbb{IR}, B = \mathbb{IR}^4$
   c) $A = \mathbb{IR}^+, B = \mathbb{IR}$
   d) $A = \mathbb{IR}^+, B = \mathbb{IR}^+$
2) Total number of sequences can be defined whose range set containing either 1 or -1 are ______.
   a) Countable infinite
   b) Uncountable infinite
   c) Two
   d) One
3) If for every $E > 0$, there exist a positive integer N does not depend on ϵ such that $|S_n - L| < \epsilon$ For all $n \geq N$ then ______
   a) All but finite number of terms of $\{S_n\}$ are equal to L
   b) No term of $\{S_n\}$ is equal to L
   c) Sequence diverges to ∞
   d) Sequence diverges to $-\infty$
4) The Series $\sum \frac{1}{n}$ is
   a) Convergent
   b) Divergent
   c) Oscillatory
   d) None of these
5) If $u(x, y) = xy$ and $v(x, y) = x + y$ then Jacobian of u and v is
   a) x
   b) y
   c) $x - y$
   d) $y - x$

Total No. of Printed Pages: 02

SUBJECT CODE NO:- B-2116
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. T.Y (Sem-V)
Examination November/December- 2022
Mathematics
Programming in C – I 505

[Time: 1:30 Hours] **[Max. Marks:40]**

Please check whether you have got the right question paper.

N.B
1) All questions are compulsory.
2) Assume the data wherever not given with justification
3) Figure to the right indicate full marks.

Q.1 A) Attempt any one: 05
a) define tokens and C tokens in C language.
b) Explain structure of C program.

B) Attempt any one: 05
c) Write a program in C for investment problem.
d) Write a program in C to add two numbers.

Q.2 A) Attempt any one: 05
a) Discuss assignment operators in C language.
b) Explain getchar function in C language

B) Attempt any one: 05
c) Write a program for printing of characters and strings.
d) Write a C program for storage classes.

Q.3 A) Attempt any one: 05
a) Explain arithmetic operators and integers operators.
b) What is explicit type conversion? Explain with example.

B) Attempt any one: 05
c) Write a program using cast to evaluate the equation.

$$Sum = \sum_{i=1}^{n} \left(\frac{1}{i}\right)$$

d) Write a C program to read integers

Q.4 Fill in the blanks: 10

1. Enumerated data type is defined as ------------

2. The assignment statement
   v op=exp;
   is equivalent to _______

3. The -----------function is used to flush out the unwanted output.

4. % s is not used to read strings with ----------

5. The operator ++ adds ---------to the operand , while ---------subtracts -----------

Total No. of Printed Pages: 2

SUBJECT CODE NO:- B-2047
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. T.Y. (Sem-V)
Examination November/December- 2022
Mathematics MAT - 502
Abstract Algebra - I

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
1. All questions are compulsory.
2. Figures to the right indicate full marks.

Q.1 A. Attempt any one of the following: 08

a. If ϕ is a homomorphism of G onto $\bar{G}$ with kernel K then prove that ${}^{G}/_{K} \approx \bar{G}$.

b. If G is a finite group and H is a subgroup of G then prove that order of H is a divisor of order of G.

B. Attempt any one of the following: 07

a. If H is a subgroup of a group G then show that $\{x \epsilon G \mid x h = hx, for\ all\ h \epsilon F\}$ is a subgroup of G.

b. Prove that the subgroup N of a group G is normal subgroup of G if and only if every left coset of N is G is a right coset of N in G.

Q.2 A. Attempt any one of the following: 08

a. If ϕ is a ring homomorphism of R into R then prove that

i) $\phi(0) = 0$

ii) $\phi(-a) = -\phi(a)$, for every $a \in R$.

b. If R is a commutative ring with unit element whose only ideals are {0} and R itself then prove that R is a field.

B. Attempt any one of the following: 07

c. If U is an ideal of a ring R then prove that [R:U]={x$\epsilon R | r.x \epsilon U$ for every $r \epsilon R$ } is an ideal of R.

d. With usual notations prove that F[x] is an integral domain.

Q.3 A. Attempt any one of the following: 05

a. Show that every subgroup of an abelian group is a normal subgroup.

b. If U is an ideal of R and 1 ϵ U then show that U=R

B. Attempt any one of the following: 05

c. Show that $x^3 - 9$ is reducible over the field of integers modulo 11.

d. If G is a group then for all a, b ϵ G prove that $(b.a)^{-1} = a^{-1}.b^{-1}$

Q.4 Choose the correct alternative and rewrite the sentence: 10

1. If o(H) divides o(G) and o(H) ≠ o(G) then ________
   a. H is a subgroup of G
   b. G is a subgroup of H
   c. G=H
   d. H may or may not be subgroup of G.

2. If G is the set of all n x n, nonsingular matrices with rational number entries then under matrix multiplication G is
   a. Finite abelian group
   b. Infinite abelian group
   c. Infinite non abelian group
   d. Finite non abelian group

3. The set of all real numbers is not a group under usual multiplication because
   a. The identity does not exist
   b. Multiplication of reals is not associative
   c. Zero has no inverse
   d. Multiplication of reals not satisfy closure property

4. If K is a subgroup of H, H is a subgroup of G and o(K)=2, o(H)=10, o(G)=20 then index of K in G is _____
   a. 2
   b. 10
   c. 20
   d. 40

5. If R is a ring then $(a-b)^2 = ------$
   a. $a^2 - 2ab + b^2$
   b. $a^2 + 2ab + b^2$
   c. $a^2 - ab - ba + b^2$
   d. $a^2 - ab + ba + b^2$

Total No. of Printed Pages:02

SUBJECT CODE NO:- B-2124
FACULTY OF SCIENCE & TECHNOLOGY
B. Sc. T.Y. (Sem-VI)
Examination November/December- 2022
Mathematics
Programming in C-II- MAT-605

[Time: 1:30 Hours] **[Max. Marks: 40]**

Please check whether you have got the right question paper.

N.B
1. All questions are compulsory.
2. Assume the data wherever not given with justification.
3. Figures to the right indicate full marks.

Q.1 A. Attempt any one 05

a. Discuss guidelines that could be followed while using indention.
b. Discuss nesting of if ________ else statements in detail.

05

B. Attempt any one

c. Write a program to evaluate power series.
$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}, 0 < x < 1.$$

d. Use switch statement to grade the students of an academic institution. (Assume data of your choice).

Q.2 A. Attempt any one 10

a. Explain do statement in detail with example.
b. Discuss in detail the jumping out of a loop.

B. Attempt any one:

c. Write a program using for loop to print the "powers of 2" table for the power 0 to 10, both positive and negative.
d. Write a program to calculate the sum of squares of all integers between 1 and 15 using if statement.

Q.3 A. Attempt any one: 10

a. Discuss compile time initialization in detail with example.
b. Explain one dimensional arrays in C language.

B. Attempt any one:

   c. Write a C program to evaluate standard deviation of given data.
   d. Write a program using a single –subscripted variable to evaluate.
      Total = $\sum_{i=1}^{30} x_i^2$

Q.4 Fill in the blanks: - 10

a. A sorted list in ________ is called ordered _____.
b. When go to statement is used many compilers generate a less _______ code
c. The ________ breaks the normal sequential execution of the program.
d. The ________ statement is an entry controlled ______ statement.
e. The ________ of the inner loop does not contain any new line character.

Total No. of Printed Pages:3

SUBJECT CODE NO:-B-2062
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. T.Y. (Sem-VI)
Examination November/December- 2022
Mathematics MAT - 602
Abstract Algebra - II

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B i) All questions are compulsory
ii) Figures to the right indicate full marks.

Q.1 (A) Attempt any one: 08

(a) Prove that if U is a vector space over F and W is a subspace of U, then there is a homomorphism of U onto U/W.

(b) If V is finite-dimensional and if W is a subspace of V, then prove that W is finite-dimensional and dim W $\leq$ dim V.

(B) Attempt any one: 07

(c) Prove that the intersection of two subspaces of a vector space V is a subspace of V.

(d) If W_1 and W_2 are subspaces of finite-dimensional vector space V over F, then show that

$$A(W_1 + W_2) = A(W_1) \cap A(W_2).$$

Q.2 (A) Attempt any one: 08

(a) Prove that a homomorphism T of an R-module M into an R-module N with kernel K(T) is an isomorphism if and only if K(T)=(O).

(b) If V is a finite-dimensional inner product space and if W is a subspace of V, then prove that $V=W+W^{\perp}$.

(B) Attempt any one: 07

(c) If S is subset of a vector space V, let $A(S) = \{f \in \hat{V} | f(s) = 0 \text{ for all } s \in S\}$. Prove that A(S)=A(L(S)), where L(S) is the linear span of S.

(d) If F is the real field and V is $F^{(3)}$, show that the Schwarz inequality implies that the cosine of an angle is of absolute value at most one.

Q.3 (A) Attempt any one: 05

(a) If S is nonempty subset of the vector space V, then prove that L(S) is a subspace of V.

(b) If u, v ∈ V and $\alpha, \beta \in F$, then prove that $\|\alpha u + \beta v\|^2 = |\alpha|^2\|u\|^2 + \alpha\bar{\beta}(u, v) + \bar{\alpha}\beta(v, u) + |\beta|^2\|v\|^2$.

Where V is an inner product space over F.

05

(B) Attempt any one:

(c) Show that in $F^{(3)}$ the vectors (1,0,0), (0,1,0) (0,0,1) are linearly independent.

(d) If V is finite-dimensional and $V_1 \neq V_2$ are in V, prove that there is an $f \in \hat{V}$ such that $f(V_1) \neq f(V_2)$.

Q.4 Choose the correct alternative: 10

(i) If V is vector space over a field F, then the subspace V itself and (O) of V are called ________.

(a) Proper subspaces (b) Improper subspaces

(c) Modules (d) None of these

(ii) If W is subspace of a vector space V over F such that dim V=8 and dim W=5, then dim A(W) ________.

(a) 13 (b) 8

(c) 3 (d) 5

(iii) The number of elements in two basis of a finite dimensional vector space is ------

(a) Equal (b) Unequal

(c) May or may not be equal (d) None of these

(iv) The dimension of a vector space R^3 over R is ________.

(a) 2 (b) 4

(c) 1 (d) 3

(v) An orthogonal set of non-zero vectors is ________.

(a) Linearly dependent

(b) Linearly independent

(c) A basis

(d) None of these

Total No. of Printed Pages:2

SUBJECT CODE NO:- B-2061
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. T.Y. (Sem-VI)
Examination November/December- 2022
Mathematics MAT-601
Real Analysis-II

[Time: 1:30 Hours] **[Max. Marks:50]**

Please check whether you have got the right question paper.

N.B
i) All questions are compulsory.
ii) Figures to the right indicate full marks.

Q.1 A. Attempt any one: 08

a) Let $\langle M_1 ,\ P_1\rangle$ and $\langle M_2 ,\ P_2\rangle$ be metric space and let $f : M_1 \to M_2$. Then prove that f is continuous on M_1 if and only $f^{-1}(G)$ is open in M_1 whenever G is open in M_2.

b) If E is any subset of a metric space M , then prove that $\bar{E}$ is closed.

B. Attempt any one: 07

c) Show that if ρ *and* σ are both metrics for a set M , then $\rho + \sigma$ is also a metric for M.

d) $If\ f : R^2 \to R^2\ is\ defined\ by\ f(\langle x , y\rangle) = (\langle y , x\rangle) \qquad (\langle x , y\rangle) \epsilon R^2,$
$show\ that\ f\ is\ continuous\ on\ R^2.$

Q.2 A. Attempt any one: 08

a) Prove that the metric space $\langle M , P\rangle$ is compact if and only if every sequence of points in M has a subsequence converging to a point in M.

b) Let f(x) be Riemann integrable in every interval and is periodic with 2π as its period , then prove that $\int_{-\pi}^{\pi} f(x)dx = \int_{-\pi}^{\pi} f(a + x)dx$ where a is any number.

B. Attempt any one: 07

c) Prove that R^2 is complete.

d) For each
n ϵ I let b_n be the subdivision $\{0 , {}^1/_n , {}^2/_n , \ldots , {}^n/_n\}$ of $[0 , 1]$. $Compute$ $\lim_{n\to\infty} L[f ; \sigma n]$ for the function $f(x) = x^2 (0 \le x \le 1)$.

Q.3 A. Attempt any one: 05

a) Let f be a continuous function from the compact metric space M_1 into the metric space M_2. Then prove that the range $f(M_1)$ of f is also compact.

b) If f is a continuous function on the closed bounded interval [a , b] , and if $\Phi'(x) = f(x)$ $(a \le x \le b)$ then prove that $\int_a^b f(x)dx = \Phi(b) - \Phi(a)$.

B. Attempt any one: 05

c) Find the Fourier series of f(x) = x in [-π, π].

d) $If\ 0 \le x \le 1\ show\ that\ \frac{x^2}{\sqrt{2}} \le \frac{x^2}{\sqrt{1+x}} \le x^2$.

Q.4 Choose the correct alternative: 10

I) The convergent sequence in a metric space has -----.
a) Unique limit c) Limit ∞
b) Distinct limit d) None of these

II) If $\langle M , P\rangle = R^1\ and\ \langle A , P\rangle = [0 , 1]$, then the open ball $B\left[0 ; \frac{1}{2}\right]\ in\ R^1$ is the interval -----.
a) $[-\frac{1}{2} , \frac{1}{2}]$ c) $(-\frac{1}{2} , \frac{1}{2}]$
b) $(0 , \frac{1}{2})$ d) $(-\frac{1}{2} , \frac{1}{2})$

III) The metric space [a, b] with absolute-value metric is -------.
a) Only totally bounded c) Bounded
b) Only complete d) Totally bounded and complete

IV) If f is a bounded function on the closed bounded interval [a , b] and σ is any subdivision of [a , b] , then $\int_{-a}^b f(x)dx = ----$.
a) $l . u . b . \cup [f , \sigma]$ c) $l . u . b . L [f ; \sigma]$
b) $g . l . b . \cup [f ; \sigma]$ d) $g . l . b . L[f , \sigma]$

V) For all n = 0 , 1 , 2 , …….., $\int_{-\pi}^{\pi} \cos^2 nx\ dx = ------$.
a) 0 b) π c) π/4 d) π^2

Total No. of Printed Pages: 2

SUBJECT CODE NO:- B-2122
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. T.Y. (Sem-VI)
Examination November/December- 2022
Mathematics
Mathematical Statistics-II – MAT -603

[Time: 1:30 Hours] **[Max. Marks:50]**

Please check whether you have got the right question paper.

N.B
1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Attempt any one: 08

a) if X and Y are random variables then prove that:

E(X+y)=E(x) + E(y),

provided that both the expectations exist

b) if X is a random variables, then prove that

$v(ax+b) = a^2 \vee (x),$

Where a and b constants.

Also prove that variance is independent of change of origin and scale.

Q.1 B) Attempt any one: - 07

c) Two unbiased dice are thrown find the expected values of the sum of numbers of points on them.

d) If x is a Poisson variate such that

$P(x=2) = gp(x=4) + 90\, P(x=6)$

Find (i) λ ii) The Mean.

Q.2 A) Attempt any one: 08

a) Find first four central moments of a binomial distribution by using moment generation function.

b) Find the moment generating function of exponential distribution.

B) Attempt any one: 07

Q.2 c) Ten coins are thrown simultaneously. Find the probability of getting at least seven heads.

d) If x and y are independent Poisson variates such that P(X=1)=P(x=2) and P(y=2)= P(y=3) find the variance of x-2y

Q.3 A) Attempt any one: 05

a) Find the median of normal distribution.

b) Prove that correlation coefficient is independent of change of origin and scale.

Q.3 B) Attempt any one: 05

a) Determine the binomial distribution for which the mean is 4 and variance 3 and find its mode..

b) If x has a Uniform distribution in [0,1] find the distribution (p.d.f) of -2logx. Identify the distribution also.

Q.4 Choose the correct alternative. 10

i. If x and y are independent then cov (x,y)=--------

a) 1 b) 0 c) -1 d) 2

ii. The mean of Poisson variate is ----------its variance.

a) Greater than b) less than c) equal to d) twice

iii. The moment generating function of gamma distribution is --------

a) $(1+t)^{\lambda}$ b) $(1-t)^{\lambda}$ c) $(1-t)^{-\lambda}$ d) $(1+t)^{-\lambda}$

iv. The mean and median of normal distribution are -------

a) The same b) Not Same c) Mean < Median d) Mean > median

v. The variance of Bernoulli distribution is -------

a) p b) pq c) q d) p-q

Total No. of Printed Pages:3

SUBJECT CODE NO:- B-2123
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. T.Y. (Sem-VI)
Examination November/December- 2022
Mathematics
Ordinary Differential Equation-II - MAT- 604

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
1) All questions are compulsory
2) Figures to the right indicate full marks

Q.1 A) Attempt any one: 08

a) Let $\phi_1, \dots, \phi_n$ be n linearly independent solutions of $L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \cdots + a_n(x)y = 0$ on an interval I. prove that if ϕ is any solution of L(y) =0 on I it can be represented in the form $\phi = c_1\phi_1 + ------ + c_n\phi_n$ where $c_1, \dots, c_n$ are constants

b) Let ϕ_1 be a solution of $L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \cdots + a_n(x)y = 0$ on an interval I and suppose $\phi_1(x) \neq 0$ on I. if $v_2, v_3, \dots, v_n$ *is any basis* on I for the solutions of the linear equation.
$\phi_1\vartheta^{(n-1)} + \cdots + \left[n\phi_1^{(n-1)} + (n-1)\phi_1^{(n-2)} + \cdots + a_{n-1}\phi_1\right]\vartheta = 0$
of order n-1 , and if
$\vartheta_k = u_k', \quad (k = 2, \dots, n)$

Then prove that $\phi_1, u_2\phi_1, \dots, u_n\phi_1$ is a basis for the solutions of $L(y) = 0$ *on I*.

B) Attempt any one 07

c) Find two linearly independent solutions of the equation
$(3x-1)^2y'' + (9x-3)y' - 9y = 0$ for $x > \frac{1}{3}$

d) Find all solutions of $y'' - \frac{2}{x^2}y = 0, (0 < x < \infty)$ given that one solution is $\phi_1(x) = x^2$

Q.2 A) Attempt any one 08

a) If $\phi_1, \dots, \phi_n$ are n solutions of $L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \cdots + a_n(x)y = 0$ on an interval I, prove that they are linearly independent if and if $W(\phi_1, \dots, \phi_n)(x) \not\supset 0$ for all x in I

b) If ϕ_1 is a solution of $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ on I and $\phi_1(x) \neq 0$ on I prove that a second solution $\phi_2(x)$ of L(y) =0 is given by

$$\phi_2(x) = \phi_1(x) \int_{x_o}^{x} \frac{1}{[\phi_1(s)]^2} \exp[-\int_{x_o}^{s} a_1(t)dt\,]\, ds$$

B) Attempt any one 07

c) Find two linearly independent power series solutions of the equation

$$y'' + 3x^2y' - xy = 0$$

d) Show that

$$\int_{-1}^{1} P_n(x)P_m(x)dx = 0\,, (n \neq m)$$

Q.3 A) Attempt any one 05

a) One solution of $xy'' - (x+1)y' + y = 0, (x > 0)$ is given as $\phi_1(x) = e^x$ find the second solution

b) Find all solutions of the equation $2x^2\, y'' + xy' - y = 0 \qquad (x > 0)$

B) Attempt any one 05

c) Compute the indicial polynomial and their roots for the equation $x^2y'' + (x + x^2)y' - y = 0$

d) Find all solutions ϕ of the form $\phi(x) = |x|^r \sum_{k=0}^{\infty} c_k x^k \qquad (|x| > 0)$
For the equations $x^2y'' + xy' + x^2y = 0$

Q.4 Choose the correct alternative 10

1) One solution of $x^2y'' - xy' + y = 0 \quad (x > 0)$ is
a) $\phi(x) = x$ b) $\phi(x) = x^2$ c) $\phi(x) = e^x$ d) $\phi(x) = e^{-x}$

2) The Bessel equation is of the form
a) $(1 - x^2)y'' - 2xy' + \propto (\propto + 1)y = 0 \;\; \propto is\ constant$
b) $x^2y'' + xy' + (x^2 - \propto^2)y = 0\,,\;\; Re \propto \geq 0$
c) $x^2y'' + axy' + by = 0$, a,b constant
d) $x^2\, y'' + 5y' + 3x^2y = 0$

3) The indicial polynomial of the equation $L(y) = x^2y'' + axy' + by = 0$ a, b constants is
a) $q(r) = r(r+1) + ar + b$
b) $q(r) = r(r-1) - ar + b$
c) $q(r) = r(r-1) + ar - b$
d) $q(r) = r(r-1) + ar + b$

4) The solutions of the equation $x^2y'' + 2xy' - 6y = 0$ for $x > 0$ are :
   a) x^2, x^3 b) x^{-2}, x^3 c) x^{-3}, x^2 d) x^{-2}, x^{-3}

5) The n-th degree Legendre polynomial $P_n(x) is\ given\ by$
   a) $\frac{1}{2^n n!}\ \frac{d^n}{dx^n}\ (x^2+1)^n$

   b) $\frac{1}{2^n n!}\ \frac{d^n}{dx^n}\ (x^2-1)^n$

   c) $\frac{1}{2^n n!}\ \frac{d^n}{dx^n} x^{2n}$

   d) $\frac{(2n)!\ \ x^{2n}}{2^n (n!)^2}$